UNITED STATES MILITARY INVOLVEMENTS SINCE 1775 MODELED AS A MARKOV PROCESS

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THESIS

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September 1970

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1970

ABSTRACT

Unclassified historical data concerning the United States involvement in military conflicts is used to model the behavioral patterns of the past as a Markov Process. The stochastic properties of the model are calculated and their usefulness as tools in the study of cost-effectiveness is discussed.

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I. A COST EFFECTIVENESS MODEL

Within the last few years the term, "cost-effectiveness" has lost its formality while extending and diversifying its applicability so that it now includes all phases of system and component design, development, production and operation. The basic concept of cost-effectiveness analysis as an analytical technique for evaluating alternative choices having economic and management implication has produced evaluation processes which are extremely sophisticated and complex, but which fail at times to provide the correct solution. The ability to make the correct decision involving an appropriate cost for a system under study remains difficult in itself and can be made virtually impossible if the decision-maker is led astray by analysis which is biased by the exclusion and/or inclusion of inappropriate cost categories. To determine the relative merits of various weapon systems it is imperative that the wartime cost categories be specifically identified and standardized. Another critical area, the mathematical relationship between cost and effectiveness, is still open to debate. This paper discusses these problems with regard to a military weapon system and develops a stochastic model which may assist in the search for correct answers.

A simple categorization of the major cost categories for a military weapon system which can be used as a focal point for developing other cost guidelines is:

Research & Development Costs

Peacetime Investment Cost

Peacetime Operating Costs

Wartime Investment Cost

and Wartime Operating Cost

In response to a similar list of cost categories TRW Systems Group has suggested, [Ref. 1], that system costs could be more explicitly related to system effectiveness if both were evaluated at the same level of military activity. For example, system wartime effectiveness is evaluated under a variety of missions and scenarios, therefore, wartime operating costs should be similarly measured at various levels of war in order to increase the preciseness of the results. Additionally, since it is common for the life-cycle of a weapon system to span periods of peacetime and various levels of military activity, estimates of system life-cycle cost should include the utilization of weighting factors applied to the various operating costs in proportion to the probability of the United States being in a state of peace or appropriate level of military involvement during a given time period. And finally, a weighting factor related to the expected frequency of war should also be used to correctly apportion the contribution of wartime investment costs to the total life-cycle costs.

As a first approximation, the military conflicts can be separated into the two categories of WAR and HALF-WAR relative to their intensity, number of casualties, etc. The stochastic properties mentioned above

can then be applied to the major cost categories to obtain a "top level" probabilistic, life-cycle cost equation for use as a guideline in final cost estimation and as a new analytical tool for gaining more insight into the resource requirements to be expected during a system's lifetime. This was done by TRW Systems Group [Ref. 1] and the resulting life-cycle cost equation was:

$$
\begin{aligned}
\text{Life-Cycle Cost} &= (\text{R\&D cost}) + (\text{Peacetime Investment cost}) \\
&+ n^{(t)}_{NW-HW} \quad (\text{Investment Cost/transition})_{NW-HW} \\
&+ n^{(t)}_{NW-W} \quad (\text{Investment Cost/transition})_{NW-W} \\
&+ n^{(t)}_{HW-W} \quad (\text{Investment Cost/transition})_{HW-W} \\
&+ \sum_{k=1}^{T} (P_{NO-WAR})_k \quad (\text{Annual Ops costs}_{PEACE})_k \\
&+ \sum_{k=1}^{T} (P_{HALF-WAR})_k \quad (\text{Annual Ops costs}_{HALF-WAR})_k \\
&+ \sum_{k=1}^{T} (P_{WAR})_k \quad (\text{Annual Ops costs}_{WAR})_k
\end{aligned}
$$

where the P's are the weighting factors based on the probability that the U. S. will be in a state of WAR, HALF-WAR, or NO-WAR in any year, k, and the n's are the number of transitions into the various states of war with both p and n being calculated over the system's total operating life span of T years. The investment cost per transition is a finite cost for war escalations and zero otherwise.

Using this approach the major categories of wartime costs can be tied explicitly to system wartime effectiveness analyses.

A mathematical model was developed by TRW Systems Group [Ref. 1] for quantitative determination of the probabilities and frequencies of war under the following conditions:

(a) The model operates in discrete time

(b) The unit of time is one year

(c) Historical data from 1900 - 1967 is considered

The purpose of this paper is to develop a mathematical model to quantitatively determine similar probabilities and frequencies, but one which operates as follows:

(a) The model operates in continuous time

(b) The unit of time is one month

(c) Historical data from 1775 - 1970 is considered.

II. HISTORICAL DATA

The military activities of the United States considered significant in the context of this paper are those listed in The Encyclopedia of Military History [Ref. 2]. References 3, 4, and 5 were useful in determining a precise commencement and/or termination date in the case of several of the conflicts being examined.

For the purpose of categorizing the various levels of military involvement, the state of WAR is defined as a shooting war with troop and equipment losses which has a distinct effect on the economy of the United States. The state of HALF-WAR is defined as a state of military involvement where troops and equipment are deployed, but either losses are relatively light or the economy of the United States as a whole is relatively unaffected. The state of NO-WAR is the state of relative peace which exists when neither a state of WAR nor a state of HALF-WAR exists.

The following interpretations were used to classify the historical data:

1. The start of a WAR during a period of HALF-WAR causes a change of state from HALF-WAR to WAR.
2. The start of a HALF-WAR during a period of WAR causes no change of state.
3. The start of a second, distinctly different, HALF-WAR during a period of existing HALF-WAR causes no change of

state, but simply permits the existing state of HALF-WAR to continue until the conclusion of the longer HALF-WAR, or a transition into a state of WAR occurs. This interpretation is taken because the calculations of cost-effectiveness in this paper depend only on the intensity of military activity and not on their theater of operation. Similar conventions apply to the states of NO-WAR and WAR, hence transitions between the same two states are not possible.

4. A change of state from HALF-WAR to WAR cannot occur as the result of a combination of concurrent HALF-WARS.

5. The month during which a change of state occurs is evaluated as a month during which the new state existed.

It is not the author's claim that this interpretation of data provides a precise categorization of the conflicts nor that the list of military activities considered is neither lacking nor in excess, but only that it provides a reasonable data base, consistent with the intent of the problem, to which an attempt can be made to fit a stochastic model.

The remaining pages of this chapter contain a complete list of the military conflicts considered, along with their classification as states of WAR (W) or HALF-WAR (HW) in accordance with the proposed scheme and a graphical representation of the resulting sequence of state transitions (Figure I).

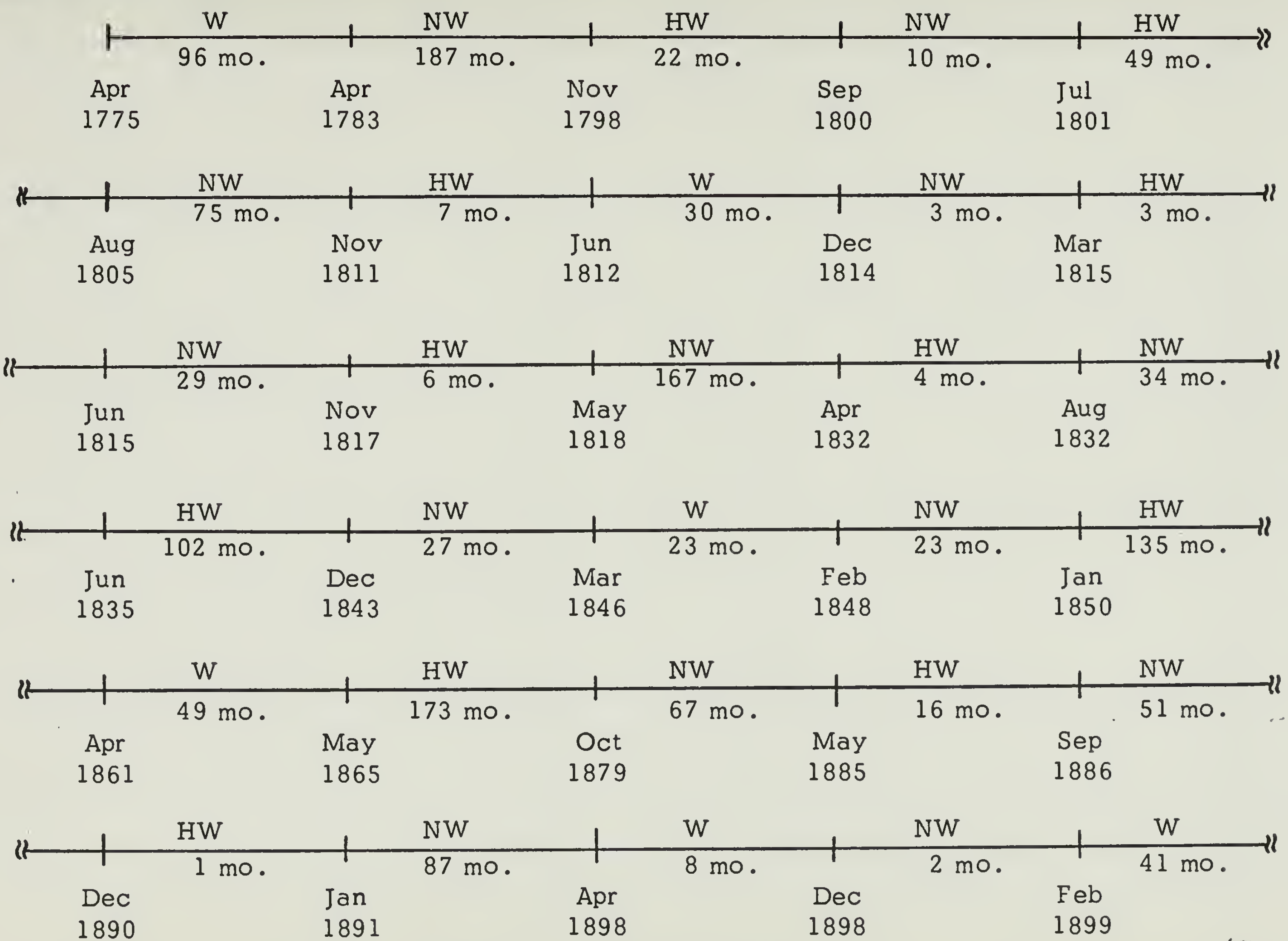
AMERICAN MILITARY ACTION 1775 - 1970

DATES	ACTION	STATE CLASSIFICATION
Apr 1775 - Apr 1783	War of the American Revolution	W
Nov 1798 - Sep 1800	Quasi War with France	HW
Jul 1801 - Aug 1805	Tripolitan War	HW
Nov 1811 - Jun 1812	Indian War against Shawnees and Friction with Britain	HW
Jun 1812 - Dec 1814	War of 1812	W
Mar 1815 - Jun 1815	American War with Algiers (Barbary Wars)	HW
Nov 1817 - May 1818	First Seminole War	HW
Apr 1832 - Aug 1832	Black Hawk War	HW
Dec 1835 - Dec 1843	Second Seminole War	HW
Jun 1835 - Apr 1836	War of Texas Independence	HW
Mar 1846 - Feb 1848	U. S. - Mexican War	W
Jan 1850 - Oct 1879	American Indian Wars	HW
Apr 1861 - May 1865	Civil War	W
May 1885 - Sep 1886	Apache War	HW
Dec 1890 - Jan 1891	Sioux War in South Dakota	HW
Apr 1898 - Dec 1898	Spanish - American War	W
Feb 1899 - Jul 1902	Guerrilla War in the Philippines	W
Jun 1900 - Oct 1900	Boxer Rebellion in China	HW
Jul 1902 - Jul 1905	Moro Campaign	HW
Nov 1903 - Nov 1903	Panamanian Revolt	HW

DATES	ACTION	STATE CLASSIFICATION
Oct 1906 - Dec 1909	Army of Cuban Pacification	HW
Jan 1912 - Jan 1913	Intervention in Honduras and Nicaragua	HW
Apr 1914 - Nov 1914	Tampico and Vera Cruz Incidents	HW
Jul 1915 - Sep 1915	Intervention in Haiti	HW
Mar 1916 - Feb 1917	Mexican Border Operations against Pancho Villa	HW
Nov 1916 - Jul 1924	Intervention in the Dominican Republic	HW
Apr 1917 - Nov 1918	World War I	W
Aug 1918 - Apr 1920	Expeditions into Northern Russia and Siberia	HW
Jan 1927 - Nov 1928	Intervention in Nicaragua	HW
Jan 1931 - Jan 1933	Intervention in Nicaragua (The Sandino Affair)	HW
Dec 1941 - Aug 1945	World War II	W
Jun 1948 - Sep 1949	Berlin Blockade	HW
Jun 1950 - Jul 1953	Korean War	W
Jul 1958 - Oct 1958	U. S. Troops to Lebanon	HW
Nov 1960 - Dec 1960	U. S. Warships Protect Guatemala and Nicaragua	HW
Dec 1961 -	U. S. Support Units Arrive in South Vietnam	HW
Oct 1962 - Nov 1962	Cuban Missile Crisis	HW
Aug 1964 -	Tonkin Gulf Resolution	W
Apr 1965 - May 1965	Intervention in Dominican Republic	HW

Sequence of Transitions and Time in State

NOT TO SCALE:



(Cont'd)

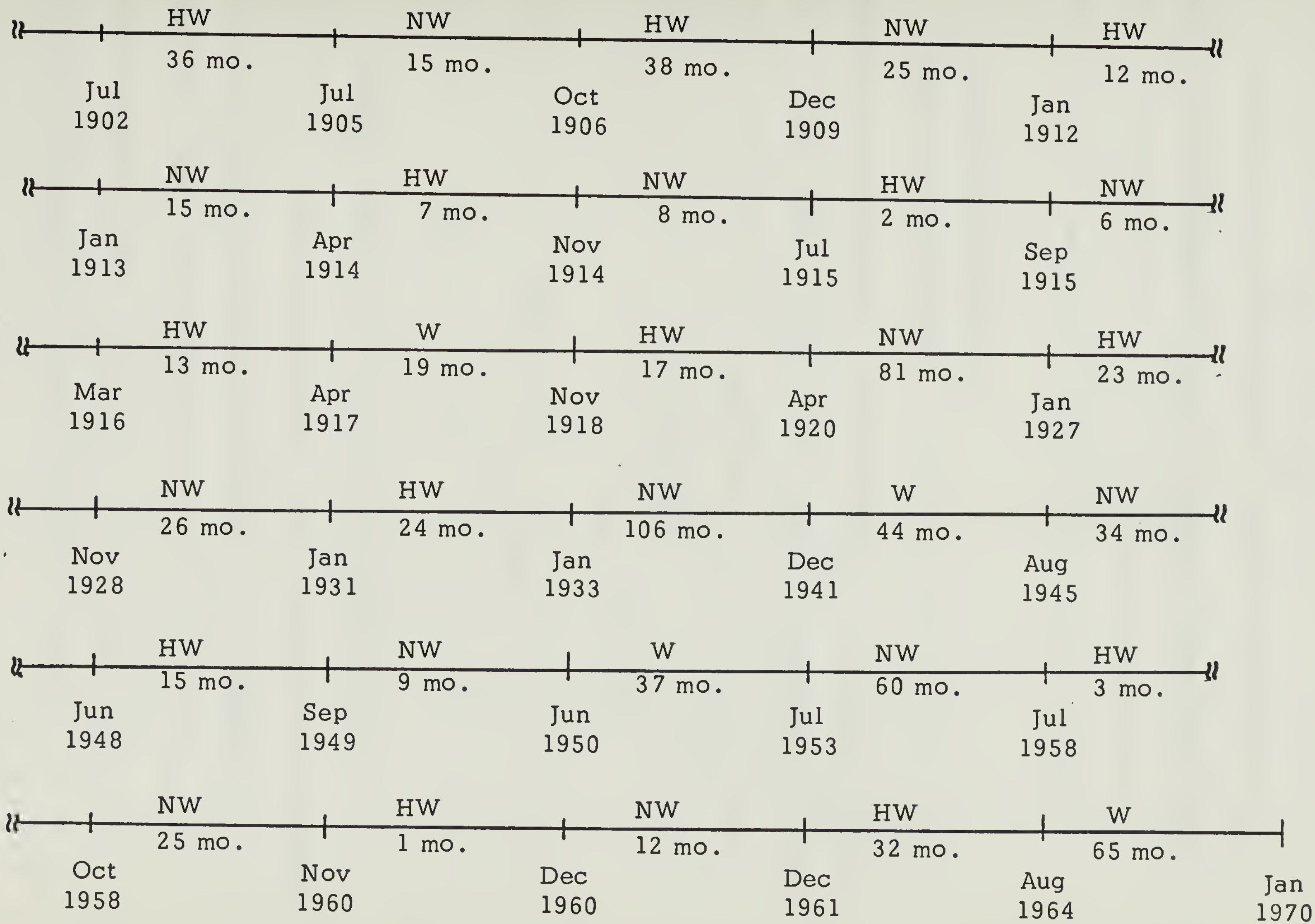

HW 36 mo.
NW 15 mo.
HW 38 mo.
NW 25 mo.
HW 12 mo.
Jul 1902
Jul 1905
Oct 1906
Dec 1909
Jan 1912
NW 15 mo.
HW 7 mo.
NW 8 mo.
HW 2 mo.
NW 6 mo.
Jan 1913
Apr 1914
Nov 1914
Jul 1915
Sep 1915
HW 13 mo.
W 19 mo.
HW 17 mo.
NW 81 mo.
HW 23 mo.
Mar 1916
Apr 1917
Nov 1918
Apr 1920
Jan 1927
NW 26 mo.
HW 24 mo.
NW 106 mo.
W 44 mo.
NW 34 mo.
Nov 1928
Jan 1931
Jan 1933
Dec 1941
Aug 1945
HW 15 mo.
NW 9 mo.
W 37 mo.
NW 60 mo.
HW 3 mo.
Jun 1948
Sep 1949
Jun 1950
Jul 1953
Jul 1958
NW 25 mo.
HW 1 mo.
NW 12 mo.
HW 32 mo.
W 65 mo.
Oct 1958
Nov 1960
Dec 1960
Dec 1961
Aug 1964
Jan 1970

FIGURE I

III. MARKOV PROCESSES

This paper uses continuous time, finite state MARKOV Processes. The definitions and special properties of these and other terms which will be used later in the construction of the mathematical model are developed at this time to provide continuity in the later sections.

A. FUNDAMENTAL DEFINITIONS

A MARKOV PROCESS (MP) is a special type of stochastic process distinguished by the property that the distribution of its future behavior, when its present state is known, is not altered by additional knowledge concerning its past behavior. Specifically, letting $\{X_t \; ; \; t = 0\}$ be the stochastic process, it's a Markov process if,

$$\Pr(X_{t_n} = x_n \mid X_{t_{n-1}} = x_{n-1}, X_{t_{n-2}} = x_{n-2}, \ldots X_{t_1} = x_1) =$$

$$\Pr(X_{t_n} = x_n \mid X_{t_{n-1}} = x_{n-1})$$

where $\quad 0 \leq t_1 \leq t_2 \leq \ldots \leq t_n$

A MARKOV CHAIN (MC) is a discrete time Markov Process, $(n = 1, 2, 3, \ldots)$, with a finite number of states. The common assumption that the transition probabilities are stationary, i.e., dependent only on the states i and j and not on the time n, is assumed in this paper. The law of motion for the process may then be described by the one step transition probabilities of going from state i to state j,

$$p_{ij} = \Pr(X_{n+1} = j \mid X_n = i)$$

A TRANSITION MATRIX is a finite array of real numbers which specify the one step transition probabilities of going from state i to state j,

$$p_{ij} = \Pr(X_{n+1} = j \mid X_n = i),$$

hence they satisfy the conditions that for every i and j,

$$p_{ij} \geqq 0$$

and $$\sum_{j=1}^{s} p_{ij} = 1$$

The classification of the states of a process become important when dealing with the asymptotic properties and also in the physical interpretation of the states. For Markov processes, the states are classified according to their limiting behavior. Given that a process is initially in state j, if the ultimate return to state j within a finite period of time is a certain event, state j is called POSITIVE RECURRENT. If a process starts in state j and subsequent visits to state j can only occur at times t, 2t, 3t ..., state j is called PERIODIC with period t. A state which is not periodic is called APERIODIC. A state is ERGODIC if it's positive recurrent and aperiodic.

In this paper all the states are ergodic and the properties developed will apply, although they are not necessarily restricted to, processes all of whose states are ergodic.

An illustrative definition of a continuous time MP with a finite state space is that it's a stochastic process which moves from one

state to another among a finite number of states with successive states visited forming a Markov chain. The process stays in any state a random length of time, the distribution function of which is exponential and dependent on the current state as well as the next state to be visited.

Define m_{ij} as the mean value of the conditional exponential distribution for the length of time spent in state i given that the next transition will be into state j, and m_i as the unconditional mean length of time spend in state i. The relationship between m_{ij} and m_i is

$$m_i = \sum_j p_{ij}\, m_{ij}$$

The unconditional mean length of time spent in state i will be a positive, finite value for every state considered in this paper, i.e., $0 < m_i < \infty$

B. LIMITING PROPERTIES

In order to obtain specific values for many of the expressions developed, it is necessary to use their limiting approximation as time approaches infinity. This limiting behavior is presented in this section.

Given that a transition into state j occurred at time $t = 0$, the expected time until the process next enters into state j is called the MEAN RECURRENCE TIME of state j and is denoted by β_j. The value of the mean recurrence time of state j is the summation, over all possible paths (θ) which begin in state j and terminate at state j with

no intervening visits to state j, of the probability of the path being taken multiplied by the expected time to traverse the path.

$$\beta_j = \sum_{(h,k)\ \varepsilon\ \theta} (\Pi\, p_{hk}) \left(\sum m_{hk} \right)$$

Over a long period of time, $(t \rightarrow \infty)$, if all states are ergodic and $m_j < \infty$ for every j, the probability of finding a Markov Process in any state j equals the ratio of the unconditional mean time spent in state j to the mean recurrence time of state j.

Specifically:

$$\lim_{t \rightarrow \infty} \Pr (X_t = j) = m_j \Big/ \beta_j$$

The above theorem with trivial alterations is proven by Smith [Ref. 6].

After a period of time $t >> \beta_j$ has elapsed, the expected number of transitions into state j is approximately the time t divided by the mean recurrence time for state j. Letting $n_j(t)$ be the number of transitions into state j during [0,t],

$$\lim_{t \rightarrow \infty} \frac{n_j(t)}{t} = \frac{1}{\beta_j} = n_j$$

The expected number of those transitions into j which came from state i is, in steady state, the number of transitions into state i times the probability of a transition from i into j.

$$n_{ij} = n_i p_{ij}$$

C. SEMI MARKOV PROCESSES

A generalization of the definition of a Markov process with regard to the distribution function of the time spent in each state results in a Semi-Markov Process (SMP).

A descriptive definition of a finite state SMP is that it is a stochastic process which moves from one state to another among a finite number of states with successive states visited forming a Markov Chain. The process stays in any state a random length of time the distribution function of which may depend on the current state as well as the next state to be visited.

Notice that the only difference between a MP and a SMP is whether or not the conditional distribution of the length of time spent in each state is exponential.

The development of the SMP which follows is based on that given by Pyke [Ref. 7].

Define $Q_{ij}(t)$ as the probability that, given the system starts in state i at time zero, the first transition is into state j and that it occurs on or before time t.

Then $Q = (Q_{ij})$, is an s x s matrix valued function defined on $(-\infty, +\infty)$, whose elements Q_{ij} are non-decreasing functions satisfying

$$(1)\ Q_{ij}(t) = 0 \qquad t \leqq 0$$

and (2) $\sum_{j=1}^{s} Q_{ij}(+\infty) = 1 \qquad 1 \leq i \leq s$

It now becomes natural to define

$$p_{ij} = Q_{ij}(+\infty)$$

and to assume that

$$p_{ij} > 0 \qquad i \neq j$$

and

$$p_{ij} = 0 \qquad i = j \quad .$$

so that the s x s matrix, $P = (p_{ij})$, becomes the transition matrix for the Markov Chain describing the sequence of states visited.

The probability of leaving state i before any time t, independent of which state you go into next, is the unconditional distribution function of the time spent in state i and is

$$Q_i(t) = \sum_{j=1}^{s} Q_{ij}(t) \qquad 1 \leq i \leq s \quad .$$

The distribution of time in state i, given that the next transition will be into state j, is simply

$$Q_{i|j}(t) = \frac{Q_{ij}(t)}{p_{ij}} \qquad i \neq j \quad .$$

The following notation is introduced to define the moments of the distributions thus far discussed. The conditional mean and conditional variance of the length of time spent in state i given that the

subsequent transition will be into state j, are respectively

$$m_{ij} = \int_{0}^{\infty} t \, d \, Q_{i|j}$$

$$\sigma^2_{ij} = \int_{0^-}^{\infty} (t - m_{ij})^2 \, d \, Q_{i|j}$$

and the mean and variance of the length of time spent in state i regardless of the subsequent transition, are

$$m_i = \int_{0}^{\infty} t \, d \, Q_i (t) \qquad = \sum_{j=1}^{s} p_{ij} \, m_{ij}$$

$$\sigma^2_i = \int_{0^-}^{\infty} (t - m_i)^2 \, d \, Q_i (t)$$

Notice that the expressions developed for the asymptotic condition as $t \to \infty$ and for the mean recurrence time are distribution free. They apply equally to a MP and SMP as long as the distribution function for the length of stay in each state has a finite mean.

D. THE ALTERNATING PROCESS

If the number of states is restricted to two and the requirements maintained that

$$p_{ij} > 0 \qquad i \neq j$$

and

$$p_{ij} = 0 \qquad\qquad i = j$$

a SMP becomes an alternating renewal process since

$$p_{ij} > 0 \text{ implies } p_{ij} = 1 \qquad\qquad i,j = 1, 2 \text{ and } i \neq j$$

In an alternating renewal process the equation for mean recurrence time reduces to the sum of the mean length of time spent in each state, because the p_{ij} equal either zero or one depending on whether or not i equals j, and is the same for both states, i.e.,

$$\beta_1 = \beta_2 = m_1 + m_2$$

The distribution function of the recurrence time is the convolution of the distribution functions which specify the length of time spent in each state. Further, the fact that

$$G_1 * G_2 (y) = G_2 * G_1 (y)$$

implies that in a two state alternating renewal process the distribution of recurrence time for each state is the same and may be called the recurrence distribution of the process.

Let Y_1 and Y_2 be random variables having distribution functions G_1 and G_2 which describe the length of time spent in states 1 and 2 respectively. The distribution of the recurrence time (G), is the distribution of the random variable Y where $Y = Y_1 + Y_2$.

$$G(y) = \int_0^y G_1 (y - x_2)\, dG_2 (x_2)$$

As a specific example, let

$$G_1(x) = 1 - e^{-\alpha x} \qquad \text{and } G_2(x) = 1 - e^{-\lambda x}$$

then

$$G(y) = G_1 * G_2 = \int_0^y [1 - e^{-\alpha(y-x)}] \lambda e^{-\lambda x} dx$$

$$= \int_0^y \lambda e^{-\lambda x} dx - \int_0^y \lambda e^{-\alpha y + \alpha x - \lambda x} dx$$

$$= \int_0^y \lambda e^{-\lambda x} dx - \lambda e^{-\alpha y} \int_0^y e^{(\alpha - \lambda)x} dx$$

$$= -e^{-\lambda x} \Big|_0^y - \frac{(\lambda e^{-\alpha y})}{(\alpha - \lambda)} e^{(\alpha - \lambda)x} \Big|_0^y$$

$$= 1 - e^{-\lambda y} - \frac{\lambda e^{-\alpha y}}{(\alpha - \lambda)} (e^{(\alpha - \lambda)y} - 1)$$

$$= 1 - e^{-\lambda y} - \frac{\lambda}{(\alpha - \lambda)} (e^{-\lambda y} - e^{-\alpha y})$$

Q.E.D.

In an alternating renewal process, the probability that the process is in state 1 at time t is the sum of the probability that state 1 was occupied initially and the length of time spent in state 1 is greater than t, and the probability that the system, independent of where it started, left state 2 at a time t', for some t' < t, and the stay in state 1 is greater than t - t'. In the limit, the value of this probability is

$$\lim_{t \to \infty} \Pr(X_t = 1) = \frac{m_1}{m_1 + m_2} = \frac{m_1}{\beta_1}$$

A formal proof using the strong law of large numbers is presented by Cox [Ref. 8].

Another interpretation of the above equation is that after a very large number of transitions, the system will have spent a proportion of time m_1 / β_1 in state 1.

Since $p_{12} = p_{21} = 1$, the expected number of transitions into state 2 from state 1 as $t \to \infty$ equals the expected number of transitions into state 1.

$$n_{12} = n_1 = t/\beta_1$$

E. STATISTICAL CALCULATIONS

When calculations are being made from sample data, the probability of a transition from state i into state j is estimated as the number of observed transition from state i into state j divided by the total number of transitions from state i. Specifically

$$p_{ij} = \frac{n_{ij}}{n_i}$$

where n_{ij} is the number of observed transitions from state i into state j.

Similarly, the mean length of time spent in any state i, conditional and unconditional, is simply the arithmetic mean length of time spent in state i from the observed data. These values are called

the sample means.

$$\bar{t}_{i|j} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} t_{i|j}^{(k)}$$

and

$$\bar{t}_{i} = \frac{1}{n_{i}} \sum_{k=1}^{n_{i}} t_{i}^{(k)}$$

IV. CURVE FITTING

The method of moments is employed for parameter estimation while attempting to fit a continuous distribution function to the data. The single parameter exponential distribution is assumed and the sample mean equated to the assumed population mean and a solution obtained for the unknown parameter.

A. THE EXPONENTIAL DISTRIBUTION

The exponential distribution function is

$$Q_{ij}(t) = 1 - e^{-\alpha_{ij} t}$$

has a mean value

$$E(t) = \frac{1}{\alpha_{ij}} = m_{ij}$$

and a variance

$$\sigma^2 = \frac{1}{\alpha_{ij}^2}$$

A convenient attribute of the exponential distribution is its "Memoryless" property. This property specifies that if the length of time spent in any state k is distributed exponentially, the remaining time in state k, given it has already been in state k for some time t_1, is a random variable with an exponential distribution independent of t_1. A proof follows:

$$Q(t) = 1 - e^{-\alpha t}$$

$$q(t) = \alpha e^{-\alpha t}$$

$$\Pr(t \geq Z) = \int_Z^{\infty} \alpha e^{-\alpha t} = e^{-\alpha Z}$$

$$\Pr(t \geq Z + t_1 \mid t \geq t_1) = \frac{\Pr(t \geq Z + t_1)}{\Pr(t \geq t_1)}$$

$$= \frac{e^{-\alpha (Z + t_1)}}{e^{-\alpha t_1}} = e^{-\alpha Z}$$

$$\Pr(t \geq Z + t_1 \mid t \geq t_1) = \Pr(t \geq Z) \qquad \text{Q.E.D.}$$

When the conditional sample mean is equated to the mean of the exponential distribution in accordance with the hypothesis, (Ho), that the conditional time in state is distributed exponentially, the results are:

$$m_{ij} = \bar{t}_{i \mid j} \rightarrow \alpha_{ij} = \frac{1}{\bar{t}_{i \mid j}}$$

B. THE KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST

The KOLMOGOROV-SMIRNOV TEST for goodness of fit, Massey [Ref. 9], is used to test the hypothesis that the data comes from an exponential distribution. The test is based on a comparison of the magnitude of the maximum difference between the set of sample values and the theoretical cumulative distribution as compared with tabulated,

critical values for the appropriate sample size at a specific level of significance. Specifically, if the assumed cumulative distribution is

$$Q_{ij}(t) = 1 - e^{-\alpha_{ij} t}$$

and the observed cumulative step-function of the sample is

$$R_{ij}(t) = \frac{r}{n_{ij}}$$

where r is the number of observations less than or equal to t; then the sampling distribution of

$$E_{ij} = \text{maximum} \left| Q_{ij}(t) - R_{ij}(t) \right|$$

is known and is independent of $Q_{ij}(t)$.

The tables associated with the Kolmogorov-Smirnov Test give critical values of the distribution of E for various sample sizes. For example, at a 0.20 level of significance, the critical value of E for $n_{ij} = 10$ is 0.322; this means that in 20 percent of the random samples of size 10, the maximum absolute deviation between the sample cumulative distribution and the population cumulative distribution will be at least 0.322. If the calculated value of E is less than this critical value there is no reason to reject the hypothesis that the sample came from the assumed distribution.

Reference 10 also compares this test with the chi-square test for goodness of fit and notes that it is superior in many cases.

V. THE TWO STATE MODEL

The two state model considers the states of WAR and NO-WAR, and will be developed separately for each of the two possible modes of data interpretation. Initially a two state model which considers each military conflict, regardless of intensity or economic implications, as a state of WAR will be formulated because this is the typical classification in studies involving cost-effectiveness. (Weapons systems employed vs weapon system not employed.) Subsequently, the interpretation of data such that all military conflicts which would not be classified as WAR in the three state model will be classified as NO-WAR.

A. THE TRANSITION MATRIX

The transition matrix for both of the two state cases is simply and uninterestingly

$$\begin{array}{c c} & \begin{array}{cc} W & NW \end{array} \\ \begin{array}{c} W \\ NW \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

B. TWO STATE MODEL (CASE I)

In case one, all the periods of conflict, regardless of intensity, are considered periods of war.

The periods of time during which a state of NO-WAR existed, as taken from Figure 1 are:

State of NW	Length (mo.)	State of NW	ength (mo.)
Apr 1783 - Nov 1798	187	Jul 1905 - Oct 1906	15
Sep 1800 - Jul 1801	10	Dec 1909 - Jan 1912	25
Aug 1805 - Nov 1811	75	Jan 1913 - Apr 1914	15
Dec 1814 - Mar 1815	3	Nov 1914 - Jul 1915	8
Jun 1815 - Nov 1817	29	Sep 1915 - Mar 1916	6
May 1818 - Apr 1832	167	Apr 1920 - Jan 1927	81
Aug 1832 - Jun 1835	34	Nov 1928 - Jan 1931	26
Dec 1843 - Mar 1846	27	Jan 1933 - Dec 1941	106
Feb 1848 - Jan 1850	23	Aug 1945 - Jun 1948	34
Oct 1879 - May 1885	67	Sep 1949 - Jun 1950	9
Sep 1886 - Dec 1890	51	Jul 1953 - Jul 1958	60
Jan 1891 - Apr 1898	87	Oct 1958 - Nov 1960	25
Dec 1898 - Feb 1899	2	Dec 1960 - Dec 1961	12

The sample mean for the state of NO-WAR,

$$\bar{t}_{NW} = \frac{1}{n_{NW}} \sum_{k=1}^{n_{NW}} t_{NW}^{(k)} = \frac{1184}{26} = 45.5 \text{ months} = 3.8 \text{ years}$$

Similar calculations for the state of WAR yield the following result:

$$\bar{t}_W = \frac{1153}{27} = 42.7 \text{ months} = 3.6 \text{ years}$$

The value of 42.70 months for the mean length of time spent in a state of WAR is slightly conservative because it assumes a transition occurred at the data end points, i.e., that a state of NO-WAR exists in March 1775 and February 1970.

The data on the length of stay in each state is assumed to have come from exponential distributions having parameters $\alpha_W = 0.0234$ and $\alpha_{NW} = 0.0220$ and mean values $m_W = 42.7$ months and $m_{NW} = 45.5$ months.

The resulting distributions, $Q_W(t)$ and $Q_{NW}(t)$ are plotted against their respective cumulative step functions from the observed date in Appendix A. In both cases the Kolmogorov-Smirnov test for goodness of fit gives no indication that the exponential assumption, (Ho), is invalid, therefore Ho is accepted.

Functionally these distributions answer the question, "Given that the United States is in a state of war, what is the probability that the war ends within two years, five years, ten years?" The answer, for this model is:

$$Q_W(t) = 1 - e^{-.0234\,t}$$

$\Pr\,(t \leqq 2 \text{ yrs}) = Q_W\,(24) = 43\%$

$\Pr\,(t \leqq 5 \text{ yrs}) = Q_W\,(60) = 75\%$

$\Pr\,(t \leqq 10 \text{ yrs}) = Q_W\,(120) = 94\%$

The mean recurrence time for the system is

$$\beta_W = \beta_{NW} = m_W + m_{NW} = 88.2 \text{ months} = 7.4 \text{ years}$$

The limiting probability that the system is in each of the two states, or the long run proportion of time spent in each state is

$$\lim_{t \to \infty} \Pr(X_t = NW) = \frac{m_{NW}}{\beta_{NW}} = \frac{45.5}{88.2} = 52\%$$

$$\lim_{t \to \infty} \Pr(X_t = \text{WAR}) = \frac{m_W}{\beta_W} = \frac{42.7}{88.2} = 48\%$$

The distribution function of the recurrence time for the system,

$$G(t) = Q_W * Q_{NW}(t)$$

where $$Q_W(x) = 1 - e^{-.0234\,x}$$

and $$Q_{NW}(x) = 1 - e^{-.0220\,x}$$

is $$G(t) = 1 - e^{-.0220\,t} - (14.6)\,(e^{-.0220\,t} - e^{-.0234\,t})$$

Using this distribution function answers can be obtained to questions such as, "Given that the U. S. is in a state of war at some time t_o, what is the probability that the U. S. will fight in a second war within ten years of time t_o ?"

$$\Pr(X_t = \text{WAR}^{(2)}, \text{ for some } t,\ t_o < t \leqq 10 \text{ yrs} \mid X_{t_o} = \text{WAR}^{(1)})$$

$$= G(120) = 77\%$$

The average number of transitions into a state of war that a system with a life cycle of thirty years should expect to experience is

$$T \frac{1}{\beta_W} = \frac{30 \text{ yrs}}{7.4 \text{ yrs}} = 4 \text{ transitions}$$

from the fact that

$$\lim_{t \to \infty} \frac{n_j(t)}{t} = \frac{1}{\beta_j} = n_j$$

C. TWO STATE MODEL (CASE II)

In case II the periods of half-war and relative peace are grouped together as the state of NO-WAR. Only the major military conflicts classified as wars in the three state model are called states of WAR in this model.

The periods of time during which a state of WAR existed, from Figure 1, are:

STATE OF WAR	LENGTH (months)
Apr 1775 - Apr 1783	96
Jun 1812 - Dec 1814	30
Mar 1846 - Feb 1848	23
Apr 1861 - May 1865	49
Apr 1898 - Dec 1898	8
Feb 1899 - Jul 1902	41
Apr 1917 - Nov 1918	19
Dec 1941 - Aug 1945	44
Jun 1950 - Jul 1953	37
Aug 1964 - Jan 1970	65

Notice that again in this model a change of state is assumed to exist at the data end points.

The sample mean from the above data, for the state of WAR is

$$\bar{t}_W = \frac{1}{n_W} \sum_{k=1}^{n_W} t_W^{(k)} = \frac{412}{10} = 41.2 \text{ months} = 3.4 \text{ years}$$

Calculations for the state of NO-WAR yield

$$\bar{t}_{NW} = \frac{1925}{9} = 213.9 \text{ months} = 17.8 \text{ years}$$

It is interesting to note that the difference in the average time spent in a state of WAR between Case I and Case II is less than two months, (42.7 mo. vs. 41.2 mo.) yet the average time spent in a state of NO-WAR varies by nearly a factor of five, (45.5 mo. vs. 213.9 mo.).

The test of the hypothesis that the data is from exponential distributions with parameters $\alpha_W = 0.0243$ and $\alpha_{NW} = 0.0047$ is contained in Appendix B. As a result of the test, the exponential hypothesis is accepted.

Given that the U. S. is in a state of NO-WAR at some time t_o, the probability that the country will escalate into a state of WAR during the next two, five or ten years is,

$$Q_{NW}(t) = 1 - e^{-.0047\, t}$$

$$\Pr(X_t = \text{WAR, for some } t,\ t_o < t \leqq 2 \text{ yrs} \mid X_{t_o} = \text{NW}) = Q_{NW}(24) = 11\%$$

$$\Pr(X_t = \text{WAR, for some } t,\ t_o < t \leqq 5 \text{ yrs} \mid X_{t_o} = \text{NW}) = Q_{NW}(60) = 24\%$$

$$\Pr(X_t = \text{WAR, for some } t,\ t_o < t \leqq 10 \text{ yrs} \mid X_{t_o} = \text{NW}) = Q_{NW}(120) = 43\%$$

The mean recurrence time for the system is

$$\beta_W = \beta_{NW} = m_W + m_{NW} = 255.1 \text{ mo} = 21.3 \text{ years}$$

The limiting probability that the system is in each of the two states, or the long run proportion of time spent in each state, is

$$\lim_{t \to \infty} \Pr(X_t = NW) = \frac{m_{NW}}{\beta_{NW}} = \frac{213.9 \text{ mo.}}{255.1 \text{ mo.}} = 84\%$$

$$\lim_{t \to \infty} \Pr(X_t = WAR) = \frac{m_W}{\beta_W} = \frac{41.2 \text{ mo.}}{255.1 \text{ mo.}} = 16\%$$

The distribution of recurrence time for the system,

$$G(t) = Q_W * Q_{NW}(t)$$

where $$Q_W(x) = 1 - e^{-.0243 x}$$

and $$Q_{NW}(x) = 1 - e^{-.0047 x}$$

is $$G(t) = 1 - e^{-.0047 t} - (0.24)\,(e^{-.0047 t} - e^{-.0243 t})$$

Given that a state of peace exists at time t_o, the probability that the U. S. will have engaged in one war and returned to a state of NO-WAR within ten years of t_o is

$$\Pr(X_{t_1} = WAR,\ X_{t_2} = NW, \text{ for some } t_1 \text{ and } t_2,\ t_o < t_1 < t_2 \leqq 10 \text{ yrs} \mid X_{t_o} = NW) = G(120) = 31\%$$

The average number of escalations into states of WAR that a system with a thirty year life cycle should expect to experience is

$$T \frac{1}{\beta_W} = \frac{30 \text{ yrs}}{21.3 \text{ yrs}} = 1 \text{ escalation}$$

from the fact that

$$\lim_{t \to \infty} \frac{n_j(t)}{t} = \frac{1}{\beta_j} = n_j \quad .$$

VI. THE THREE STATE MODEL

The interpretation of data for the three state model is presented in Figure 1.

A. STATE TRANSITIONS

The conditional sample mean time in state,

$$\bar{t}_{i|j} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} t_{i|j}^{(k)}$$

the corresponding values of the exponential parameter,

$$(\alpha_{ij}) = \frac{1}{\bar{t}_{i|j}}$$

and the unconditional sample mean time in state

$$t_i = \sum_{j=1}^{s} p_{ij}\, t_{i|j}$$

will be calculated and the three state transition matrix developed.

It must be noted that the convenient assumption that a transition occurs at the data end point of January 1970 is of no value in this model because knowledge of the next state is not a certain event. The result of this conditional dependence on the next state visited is that the unconditional mean time spent in the state of WAR will be conservative since it was computed from

$$m_W = \bar{t}_W = \sum_{j=1}^{s} p_{ij} \, t_{i|j}$$

and neither $t_{W|NW}$ nor $t_{W|HW}$ considers the length of the Vietnam conflict which has as a minimum length a value which would cause an increase in the computed average time spent in a state of WAR.

NW - HW	
Nov 1798	187
Jul 1801	10
Nov 1811	75
Mar 1815	3
Nov 1817	29
Apr 1832	167
Jun 1835	34
Jan 1850	23
May 1885	67
Dec 1890	51
Oct 1906	15
Jan 1912	25
Apr 1914	15
Jul 1915	8
Mar 1916	6
Jan 1927	81
Jan 1931	26
Jun 1948	34
Jul 1958	60
Nov 1960	25
Dec 1961	12
	953 mo.

NW - W	
Mar 1846	27
Apr 1898	87
Feb 1899	2
Dec 1941	106
Jun 1950	9
	231 mo.

$p_{NW-W} = 5/26 = 0.19$

$m_{NW-W} = \bar{t}_{NW|W} = 46.2$ mo.

$= 3.9$ yrs.

$\alpha_{NW-W} = 0.0216$

$m_{NW} = 45.5$ mo. $= 3.8$ yrs.

$p_{NW-HW} = 21/26 = 0.81$

$m_{NW-HW} = \bar{t}_{NW|HW} = 45.4$ mo. $= 3.8$ yrs.

$\alpha_{NW-HW} = 0.0220$

HW - NW	
Sep 1800	22
Aug 1805	49
Jun 1815	3
May 1818	6
Aug 1832	4
Dec 1843	102
Oct 1879	173
Sep 1886	16
Jan 1891	1
Jul 1905	36
Dec 1909	38
Jan 1913	12
Nov 1914	7
Sep 1915	2
Apr 1920	17
Nov 1928	23
Jan 1933	24
Sep 1949	15
Oct 1958	3
Dec 1960	1
	554 mo.

$p_{HW-NW} = 20/24 = 0.83$

$m_{HW-NW} = \bar{t}_{NW|HW} = 27.7$ mo. $= 2.3$ yrs.

$\alpha_{HW-NW} = 0.0361$

HW - W	
Jun 1812	7
Apr 1861	135
Apr 1917	13
Aug 1964	32
	187 mo.

$p_{HW-W} = 4/24 = 0.17$

$m_{HW-W} = \bar{t}_{HW|W} = 46.8$ mo.

$= 3.9$ yrs.

$\alpha_{HW-W} = 0.0214$

$m_{HW} = 30.1$ mo. $= 2.5$ yrs.

W - NW	
Apr 1783	96
Dec 1814	30
Feb 1848	23
Dec 1898	8
Aug 1945	44
Jul 1953	37
	238 mo.

W - HW	
May 1865	49
Jul 1902	41
Nov 1918	19
	109 mo.

$p_{W-NW} = 6/9 = .67$

$m_{W-NW} = \bar{t}_{W|NW} = 39.7$ mo.

$= 3.3$ yrs.

$\alpha_{W-NW} = 0.0252$

$p_{W-HW} = 3/9 = .33$

$m_{W-HW} = \bar{t}_{W|HW} = 36.3$ mo.

$= 3.0$ yrs.

$\alpha_{W-HW} = 0.0275$

$$m_W = 38.6 \text{ mo.} = 3.2 \text{ yrs.}$$

The resulting transition matrix is

$$\begin{array}{c c} & \begin{array}{ccc} NW & HW & W \end{array} \\ \begin{array}{c} NW \\ HW \\ W \end{array} & \begin{bmatrix} 0 & .81 & .19 \\ .83 & 0 & .17 \\ .67 & .33 & 0 \end{bmatrix} \end{array}$$

The distribution functions $Q_{ij}(t)$ and $R_{ij}(t)$ are plotted for each i and j, and the value of E_{ij} is determined in Appendix C. In no case is any cause for rejection of the hypothesis found, therefore it is accepted that the conditional time in state is exponentially distributed making the Semi-Markov process actually a Markov process.

The matrix of conditional transition distributions, $Q = (Q_{ij})$, is

$$\begin{array}{c|ccc} & NW & HW & W \\ \hline NW & 0 & .81\,(1-e^{-.0220t}) & .19\,(1-e^{-.0216t}) \\ HW & .83\,(1-e^{-.0361t}) & 0 & .17\,(1-e^{-.0214t}) \\ W & .67\,(1-e^{-.0252t}) & .33\,(1-e^{-.0275t}) & 0 \end{array}$$

and the unconditional distribution of time spent in each state,

$$Q_i(t) = \sum_j Q_{ij}(t)$$

is:

$$Q_{NW}(t) = 1 - (.81)e^{-.0220t} - (.19)e^{-.0216t}$$

$$Q_{HW}(t) = 1 - (.83)e^{-.0361t} - (.17)e^{-.0214t}$$

$$Q_W(t) = 1 - (.67)e^{-.0252t} - (.33)e^{-.0275t}$$

The following questions are typical of those which can be answered using these distribution functions.

Given that a state of HALF-WAR currently exists, what is the probability that an escalation into a state of WAR will occur within the next five years and prior to a de-escalation into a state of peace? Statistically the question asks for the product of the probability that the next transition is from HW into WAR and the probability that the transition occurs within the next five years.

The answer is obtained by simply evaluating the appropriate element of the Q matrix, at a time equal to five years. In this case

$.17\,(1 - e^{-.0214t}) = 12\%$ at $t = 5$ years.

Given a state of WAR exists, what is the probability that a de-escalation will occur within the next five years?

$$Q_W(t) = 1 - (.67)e^{-.0252t} - (.33)e^{-.0275t}$$

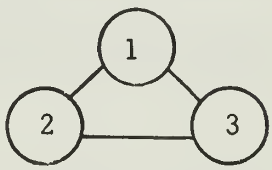
$$Q_W(60) = 79\%.$$

B. MEAN RECURRENCE TIMES

The equation for mean recurrence time,

$$\beta_j = \sum_{(h,k)\,\varepsilon\,\theta} (\prod p_{hk}) (\sum m_{hk})$$

may be described in a form more responsive to computational manipulations by attempting a "brute force" approach when the number of states is sufficiently small to permit this tactic.



For example, in a basic three state model, by beginning in state 1 and considering initially only those paths whose first transition is into state 2, it is possible to write an expression for the average length of time it would take to traverse each possible path, from state 1 back into state 1, times the probability of such a path being taken. For two, three and four transitions, the expressions are respectively:

$$p_{12}\, p_{21}\, (m_{12} + m_{21})$$

$$p_{12}\, p_{23}\, p_{31}\, (m_{12} + m_{23} + m_{31})$$

$$p_{12}\, (p_{23}\, p_{32})\, p_{21}\, (m_{12} + m_{23} + m_{32} + m_{21})$$

It will soon be evident that the paths being described which contain an even number of transition form the series

$$\sum_{k=0}^{\infty} p_{12}\, p_{21}\, (p_{23}\, p_{32})^k \left[m_{12} + m_{21} + k(m_{23} + m_{32})\right]$$

and those containing an odd number of transitions form the series

$$\sum_{k=0}^{\infty} p_{12}\, p_{23}\, p_{31}\, (p_{32}\, p_{23})^k \left[m_{12} + m_{23} + m_{31} + k\,(m_{32} + m_{23})\right]$$

Notice that this approach applies because $p_{ii} = 0$.

Because of symmetry in the model the paths from state 1 which go initially into state 3 form identical series with the subscripts 2 and 3 reverse. Therefore, all possible first passage paths from state 1 may be explicitly counted.

As stated previously, the product of the average length of time spent traversing each of these paths and the probability that such a path will be selected, when summed over all possible paths becomes the mean recurrence time (β), which in this instance is

$$\beta_1 = \sum_{k=0}^{\infty} p_{12}\, p_{21}\, (p_{23}\, p_{32})^k \left[m_{12} + m_{21} + k\,(m_{23} + m_{32})\right]$$

$$+ \sum_{k=0}^{\infty} p_{12}\, p_{23}\, p_{31}\, (p_{32}\, p_{23})^k \left[m_{12} + m_{23} + m_{31} + k(m_{32} + m_{23})\right]$$

(cont'd)

$$+ \sum_{k=0}^{\infty} p_{13}\, p_{31}\, (p_{32}\, p_{23})^k \left[m_{13} + m_{31} + k(m_{32} + m_{23}) \right]$$

$$+ \sum_{k=0}^{\infty} p_{13}\, p_{32}\, p_{21}\, (p_{23}\, p_{32})^k \left[m_{13} + m_{32} + m_{21} + k(m_{23} + m_{32}) \right]$$

the term, $(p_{23}\, p_{32})$, will always be less than one, hence upon expanding the above equation, each term may be recognized as either a geometric series or a simple variation thereof, both of which have limits.

Specifically,

$$\sum_{k=0}^{\infty} a\, p^k = \frac{a}{(1 - p)} \qquad p < 1$$

$$\sum_{k=0}^{\infty} a\, k\, p^k = \frac{a\, p}{(1 - p)^2} \qquad p < 1$$

The equation for mean recurrence time becomes

$$\beta_1 = \left(\frac{1}{1 - p_{23}\, p_{32}} \right) \Big[p_{12}\, p_{21}\, (m_{12} + m_{21}) + p_{12}\, p_{23}\, p_{31} (m_{12} + m_{23} + m_{31})$$

$$+ p_{13}\, p_{31}\, (m_{13} + m_{31}) + p_{13}\, p_{32}\, p_{21}\, (m_{13} + m_{32} + m_{21}) \Big]$$

$$+ \frac{p_{23}\, p_{32}}{(1-p_{23}\, p_{32})^2} \left[m_{32} + m_{23} \right] \left[p_{12}\, p_{21} + p_{12}\, p_{23}\, p_{31} + p_{13}\, p_{31} + p_{13} p_{32}\, p_{21} \right]$$

When this equation is used for the calculation of mean recurrence times the following results are obtained:

$$\beta_W = 252.5 \text{ months} = 21 \text{ years}$$

$$\beta_{HW} = 94.7 \text{ months} = 7.9 \text{ years}$$

$$\beta_{NW} = 87.4 \text{ months} = 7.3 \text{ years}$$

The interpretation placed on these values is that over a great many years the average length of time the United States spends between states of WAR is twenty-one years, between states of NO-WAR approximately seven years, etc.

A comparison can be made between the mean recurrence times suggested by the model and the arithmetic average recurrence times from the data, however, it should be noted that very close agreement should only be expected between the mean recurrence times for WAR. The end points of the data are both states of WAR therefore the arithmetic average time between states of WAR involves no assumptions and should agree with the value suggested by the model.

The longest interval between states of HALF-WAR is the conservative 283 months which occurs before the first HALF-WAR. Current events indicate that the length of the Vietnam conflict is also a conservative value and when applied as a mean recurrence time for HALF-WAR it becomes even more conservative because it may be followed by a period of NO-WAR before a recurrence of HALF-WAR. A similar argument applies to the state of NO-WAR; therefore it is expected that the arithmetic average recurrence time for HW and NW will be less than the value suggested by the model.

The values of mean recurrence time compare as follows:

State	calculated value from the model	arithmetic average value from the data
WAR	253 mo.	214 mo.
HALF-WAR	95 mo.	64 mo.
NO-WAR	87 mo.	43 mo.

The large disparity between the calculated and sample value for the state of NO-WAR is a result of the fact that of the ten sample points where the length of time in a state was six months or less, seven appear as sample values of recurrence time for the state of NO-WAR. Perhaps the tactic of "display of strength" such as sending warships to protect Guatemala and Nicaragua in late 1960 or the Cuban Missile Crisis in 1962 should not be considered as states of HALF-WAR because of their brevity and the fact that weapons per se were not expended. This would certainly decrease the variance between the two values in question. Yet the possibility that the appearance of the majority of the small sample values as intervals between states of NO-WAR may be classified statistically as an unusual sample cannot be dismissed. The sensitivity of the model to changes in data base is not investigated in the paper, however it is assumed, as an alternative to declaring the model a misfit, that a less than optimal method of historical data classification plus the occurrence of a statistical rare event account for the difference between the two values.

C. PROBABILITY OF STATE OCCUPATION

The limiting probability of finding the system in each of the three states,

$$\lim_{t \to \infty} \Pr(X_t = j) = \frac{m_j}{\beta_j}$$

is:

$$\lim_{t \to \infty} \Pr(X_t = \text{WAR}) = \frac{38.6}{252.5} = 15\%$$

$$\lim_{t \to \infty} \Pr(X_t = \text{HW}) = \frac{30.1}{94.7} = 32\%$$

$$\lim_{t \to \infty} \Pr(X_t = \text{NW}) = \frac{45.5}{87.4} = 52\%$$

The probabilities do not sum to one because the mean time in state was calculated as the sum of the conditional mean times and therefore precluded the use of all the data. The difference of 1% is considered insignificant.

D. NUMBER OF ESCALATIONS INTO WAR STATES

Employing the limiting probability that

$$\lim_{t \to \infty} \frac{n_j(t)}{t} = \frac{1}{\beta_j} = n_j$$

and the fact that

$$n_{ij} = n_i \, p_{ij}$$

the following type of questions can be answered.

Given a weapon system with a thirty year life cycle, the average number of war escalations, of each type, it should expect to witness is

$$n_{NW-HW}\,T = n_{NW}\,p_{NW-HW}\,T = \frac{1}{7.3}\,(.81)\,(30) = 3 \text{ escalations}$$

$$n_{NW-W}\,T = n_{NW}\,p_{NW-W}\,T = \frac{1}{7.3}\,(.19)\,(30) = 1 \text{ escalation}$$

$$n_{HW-W}\,T = n_{HW}\,p_{HW-W}\,T = \frac{1}{7.9}\,(.17)\,(30) = 1 \text{ escalation}$$

E. LIFE CYCLE COST EQUATION

Using the limiting values obtained for the three state model, the long run, average life cycle cost equation of chapter I can be written more explicitly for a weapon system with a life cycle of T years, as:

$$\begin{aligned}
\text{Life Cycle Cost} = {} & (\text{R\&D cost}) + (\text{Peacetime Investment cost}) \\
& + (.111\,T)\,(\text{Investment cost/transition})_{NW-HW} \\
& + (.026\,T)\,(\text{Investment cost/transition})_{NW-W} \\
& + (.021\,T)\,(\text{Investment cost/transition})_{HW-W} \\
& + (.52\,T)\,(\text{Annual Ops. Cost}_{PEACE}) \\
& + (.32\,T)\,(\text{Annual Ops. Cost}_{HALF-WAR}) \\
& + (.15\,T)\,(\text{Annual Ops. Cost}_{WAR})
\end{aligned}$$

In this form it is conceivable that such an equation could be of value to a decision maker.

VII. COMPARISON OF RESULT FROM EACH MODEL

The following is a tabulation of the important results from the three cases investigated. All times are in months.

	2 - State		3 - State
	Case I (HW=W)	Case II (HW=NW)	
m_{NW}	45.5	213.9	45.5
m_{HW}			30.1
m_{W}	42.7	41.2	38.6
β_{NW}	88.2	255.1	87.4
β_{HW}			94.7
β_{W}	88.2	255.1	252.5
$\lim_{t \to \infty} Pr(X_t=NW)$	.52	.84	.52
$\lim_{t \to \infty} Pr(X_t=HW)$			.32
$\lim_{t \to \infty} Pr(X_t=W)$	.48	.16	.15

The results provided by Case I of the two state model compare more favorably with the results of the three state model, (except for the mean recurrence time of war) than those of Case II, however it should not be assumed from this that Case I is to be preferred to Case II.

If conditions necessitated the use of a two state model the nature of the problem and type of solution desired would indicate the more preferable case.

VIII. CONCLUSION

In the author's opinion, the area most susceptible to error is that of data identification and classification. Further it is felt that this error will be minimized if the historical data utilized is restricted to conflicts of the specific type or style of combat being considered in the problem at hand. For example, in a cost-effectiveness study for the selection of the best among several amphibious landing vehicles the historical data should be restricted to conflicts which included amphibious assaults.

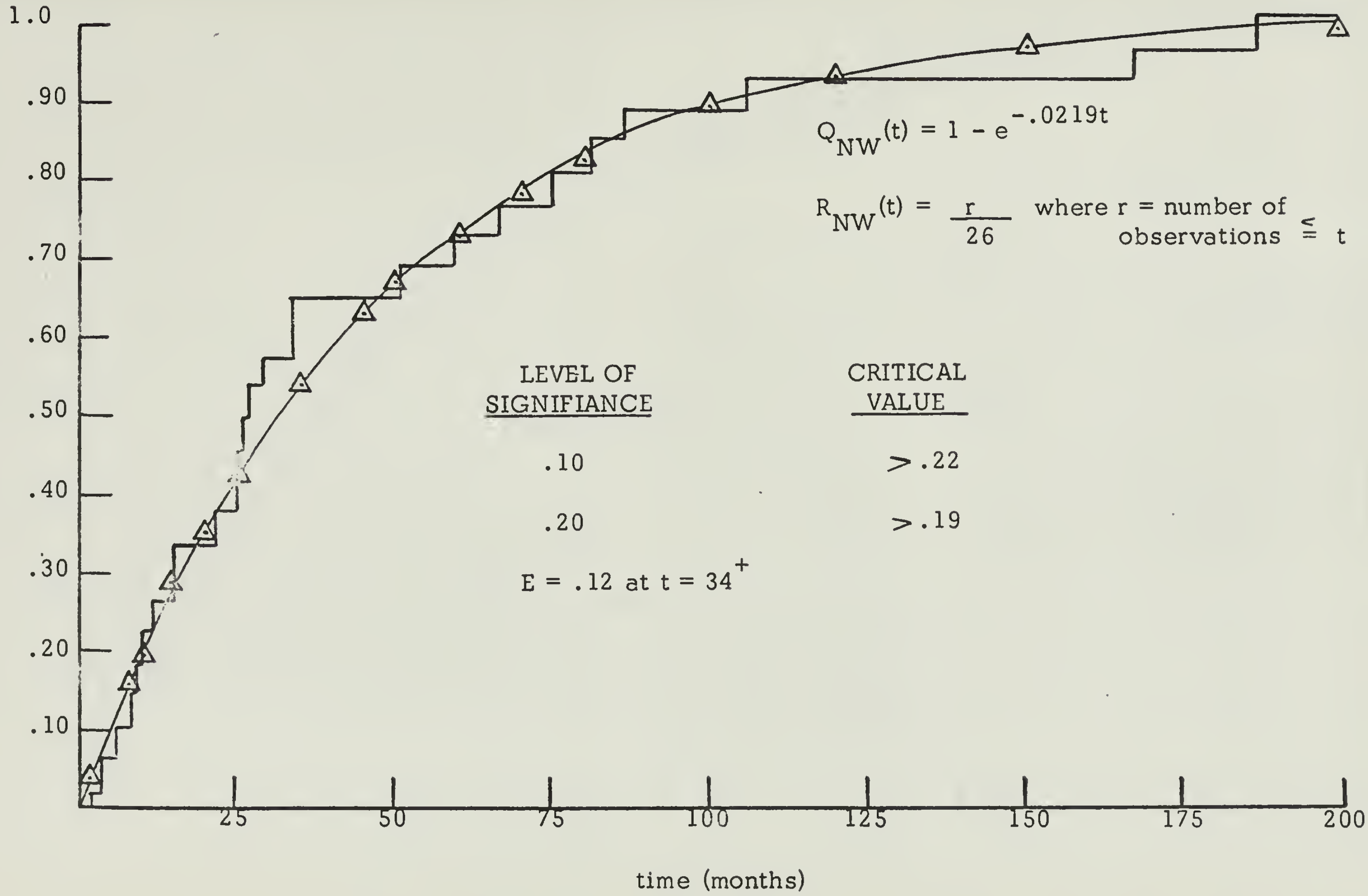
The reader is warned against using the model to predict a specific event or to answer questions such as, "Given the U. S. is in a state of WAR in January 1970, what is the probability that the war will end by January 1973?" The values obtained from the model are the average values to be expected over a long period of time and their use in predicting individual occurrences should be avoided.

The simplicity of the model, as indicated by the fact that all the calculations can be done by hand, should encourage its use as a tool in cost-effectiveness methodology and encourage an extension of the technique to other applications.

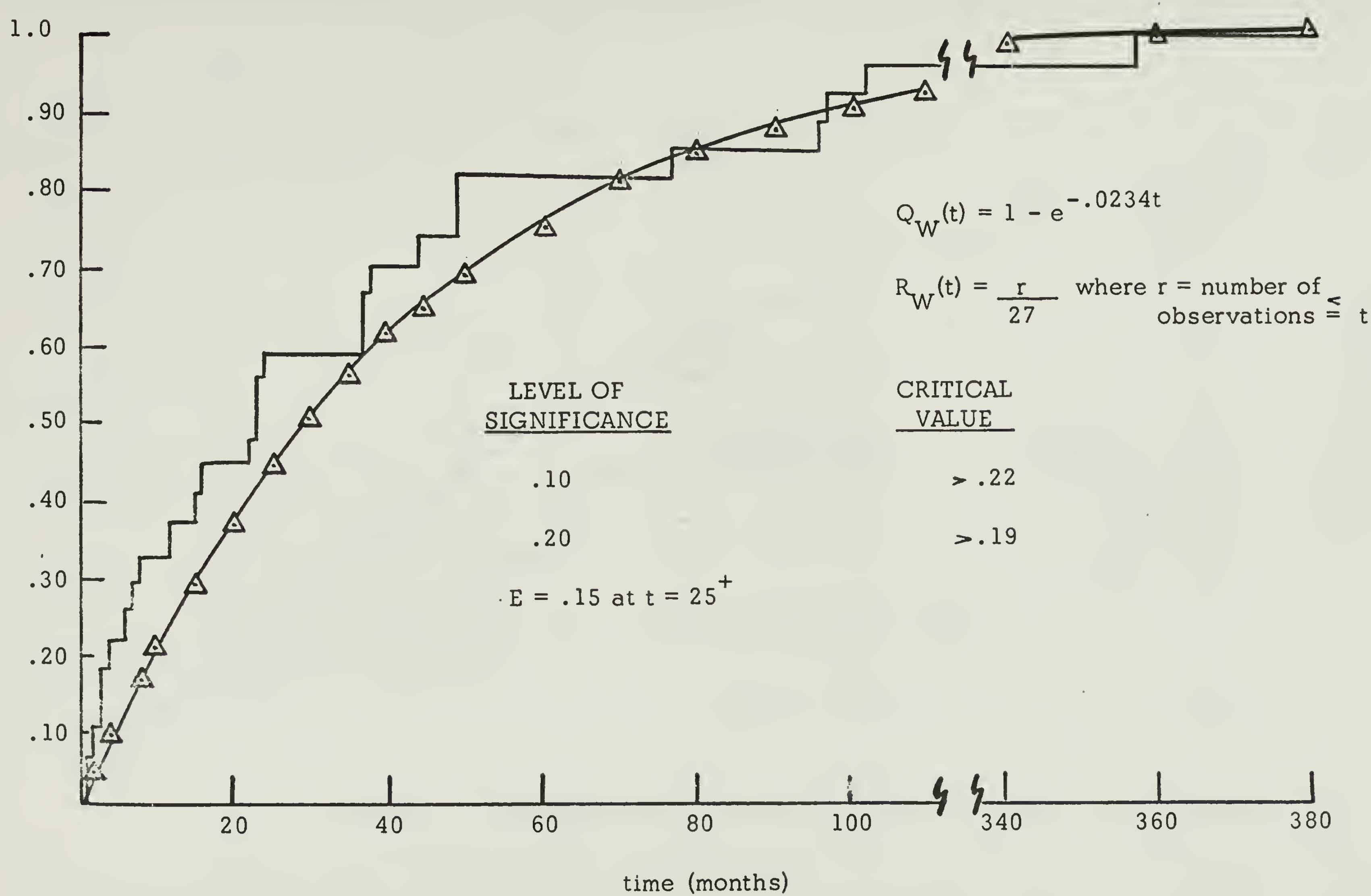
APPENDIX A

2 STATE MODEL CASE I (HW = W)

NO WAR



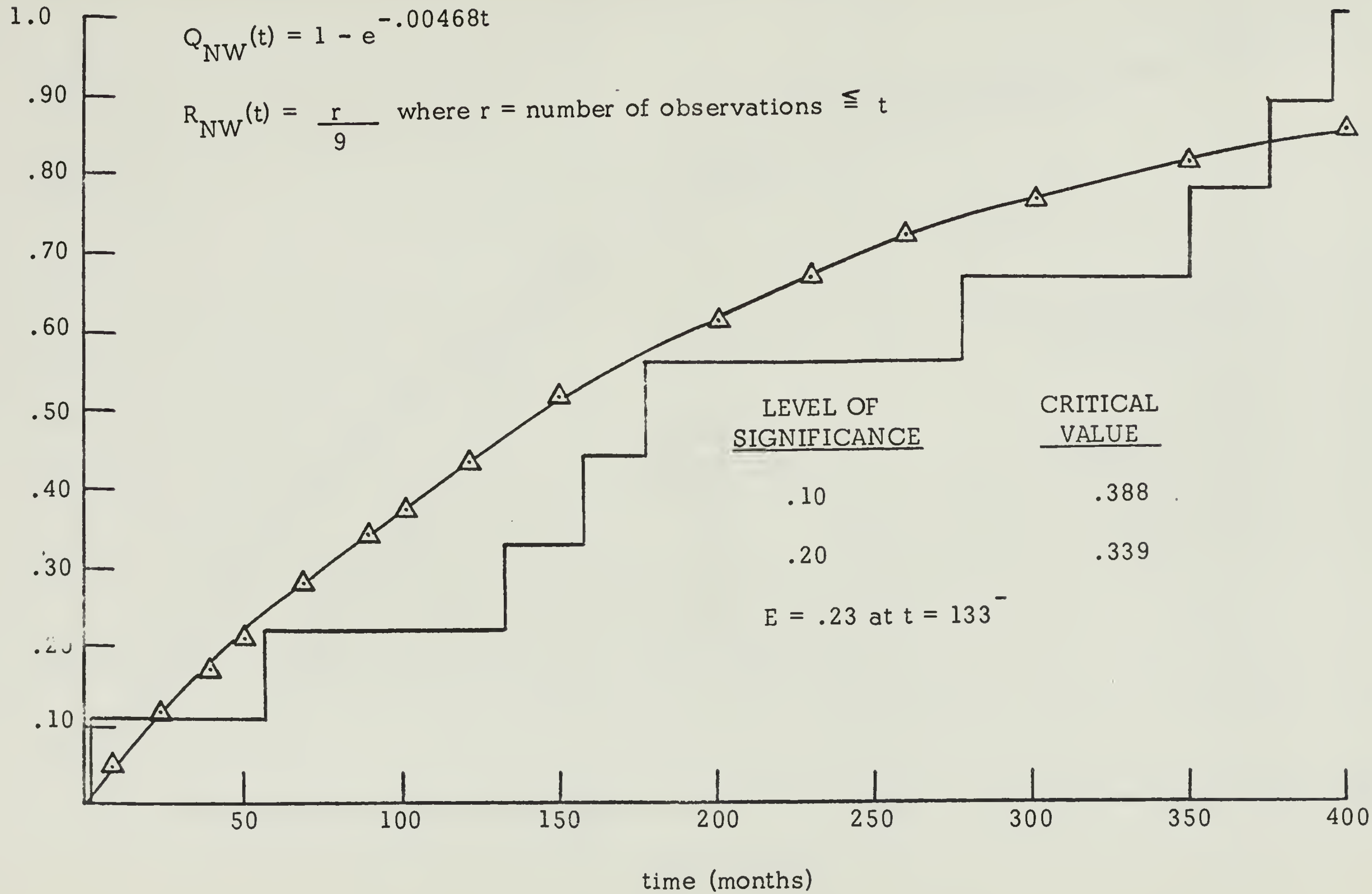
2 STATE MODEL CASE I (HW = W)
WAR



APPENDIX B

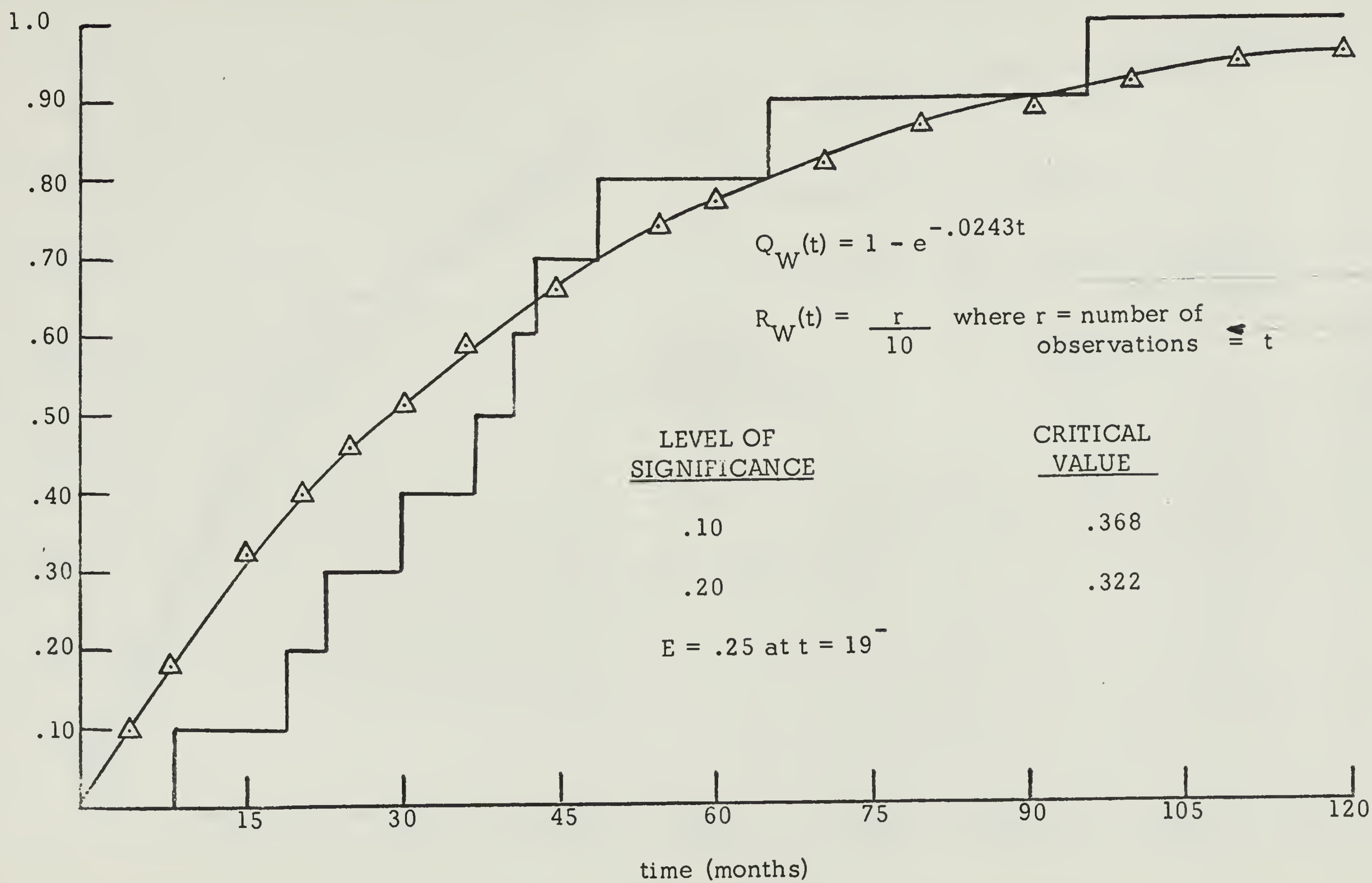
2 STATE MODEL CASE II (HW = NW)

NO WAR



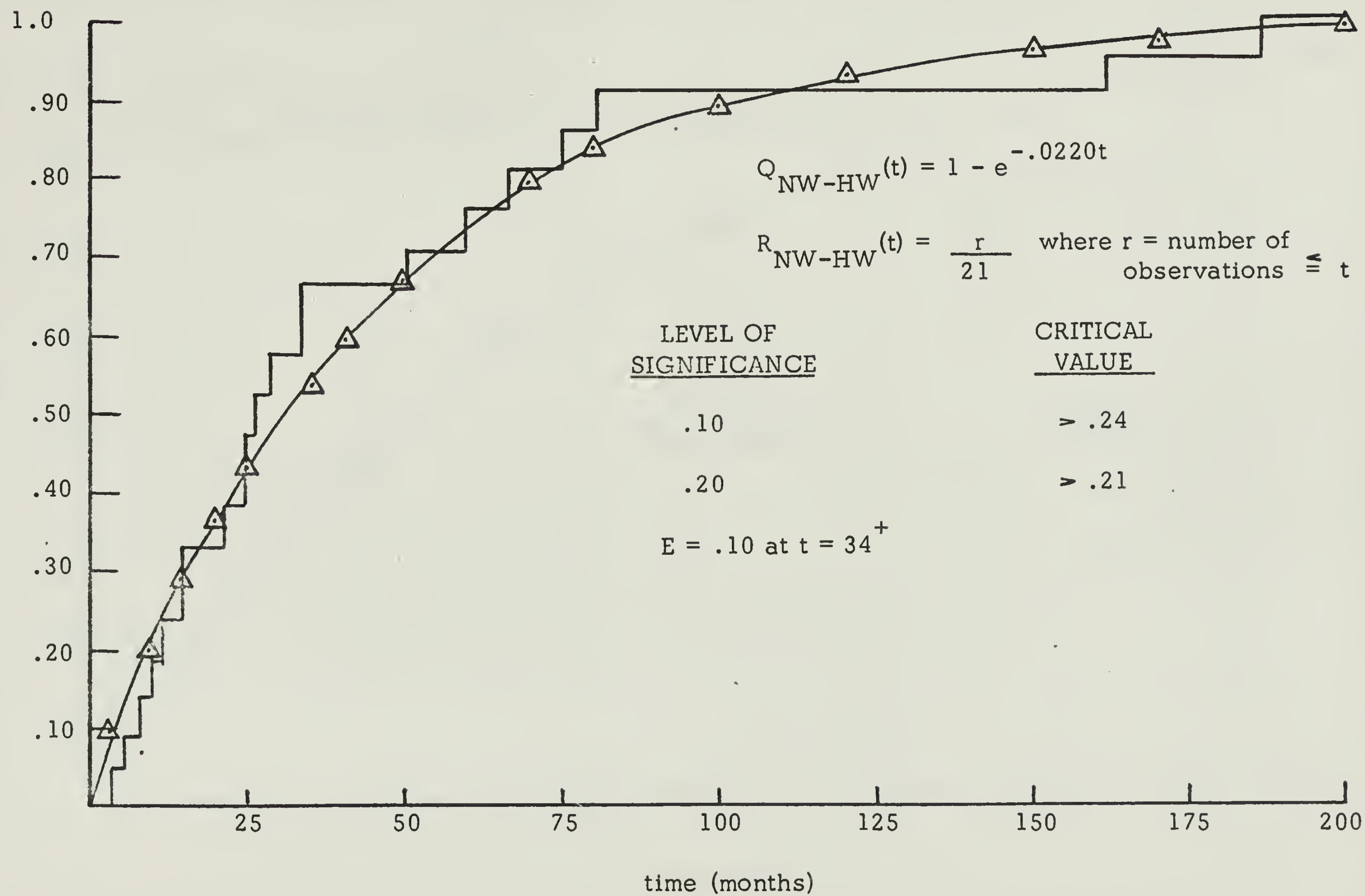
2 STATE MODEL CASE II (HW = NW)

WAR

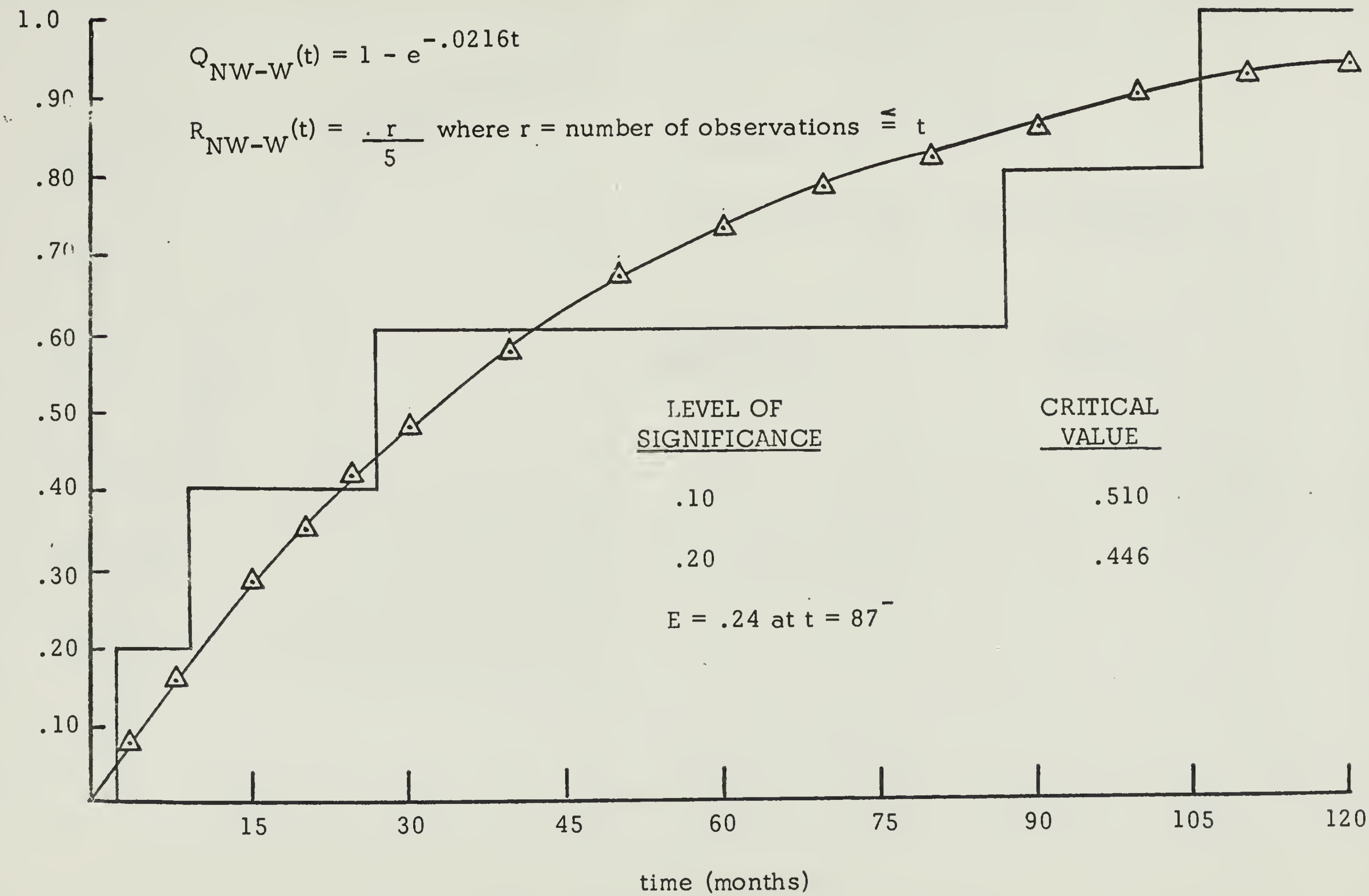


APPENDIX C

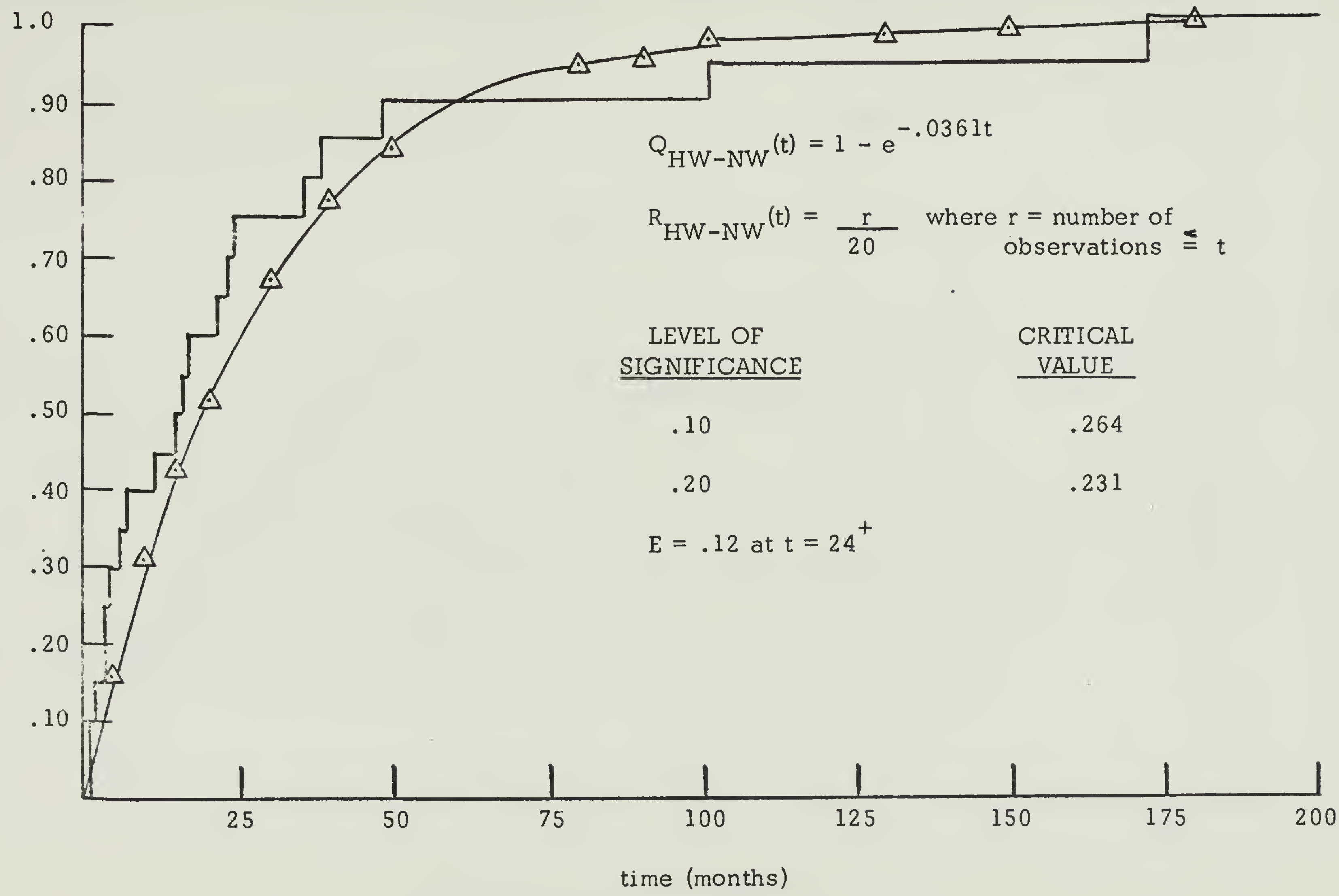
3 STATE MODEL
NO WAR - HALF WAR



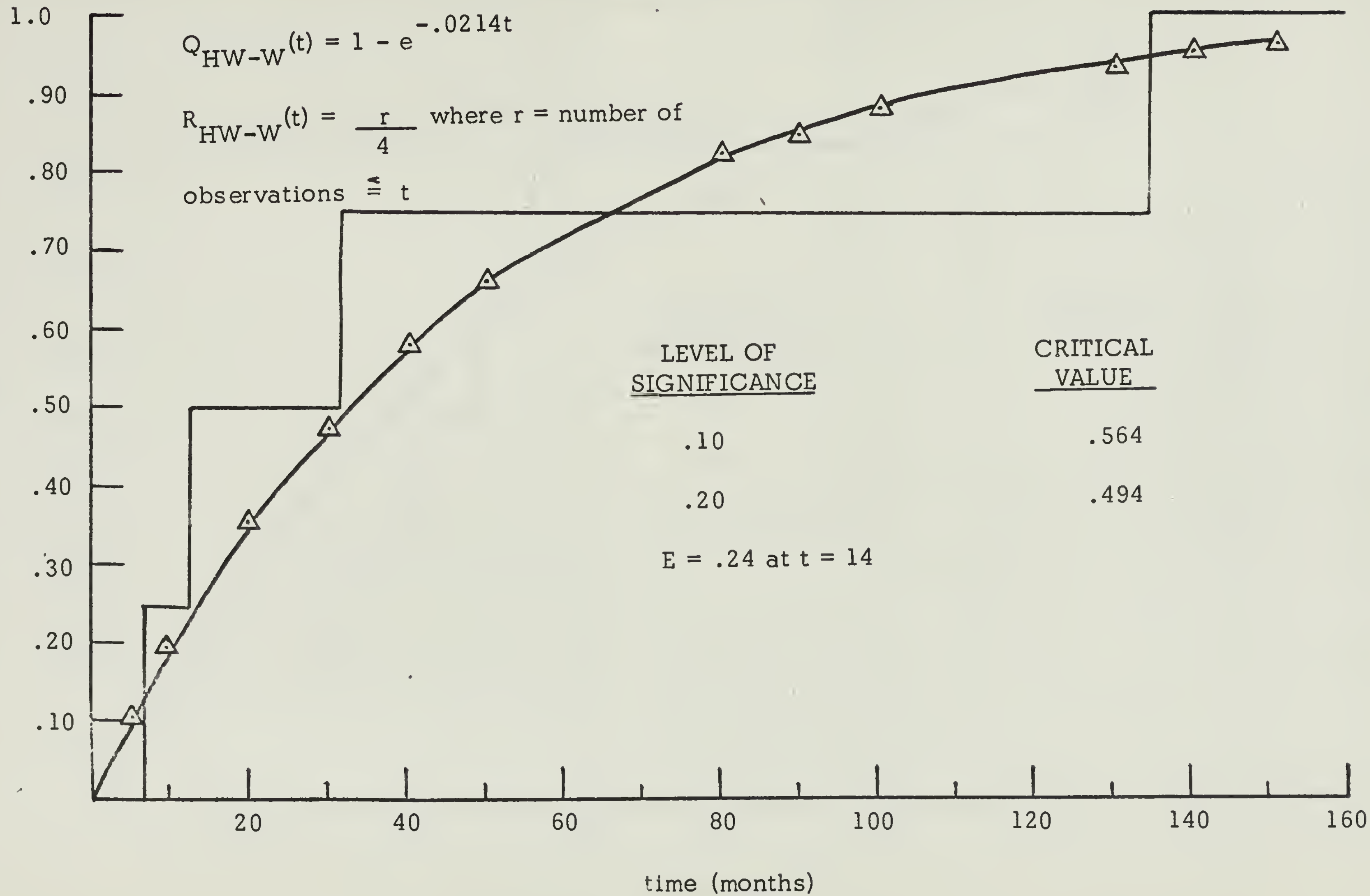
3 STATE MODEL
NO WAR - WAR



3 STATE MODEL
HALF WAR - NO WAR

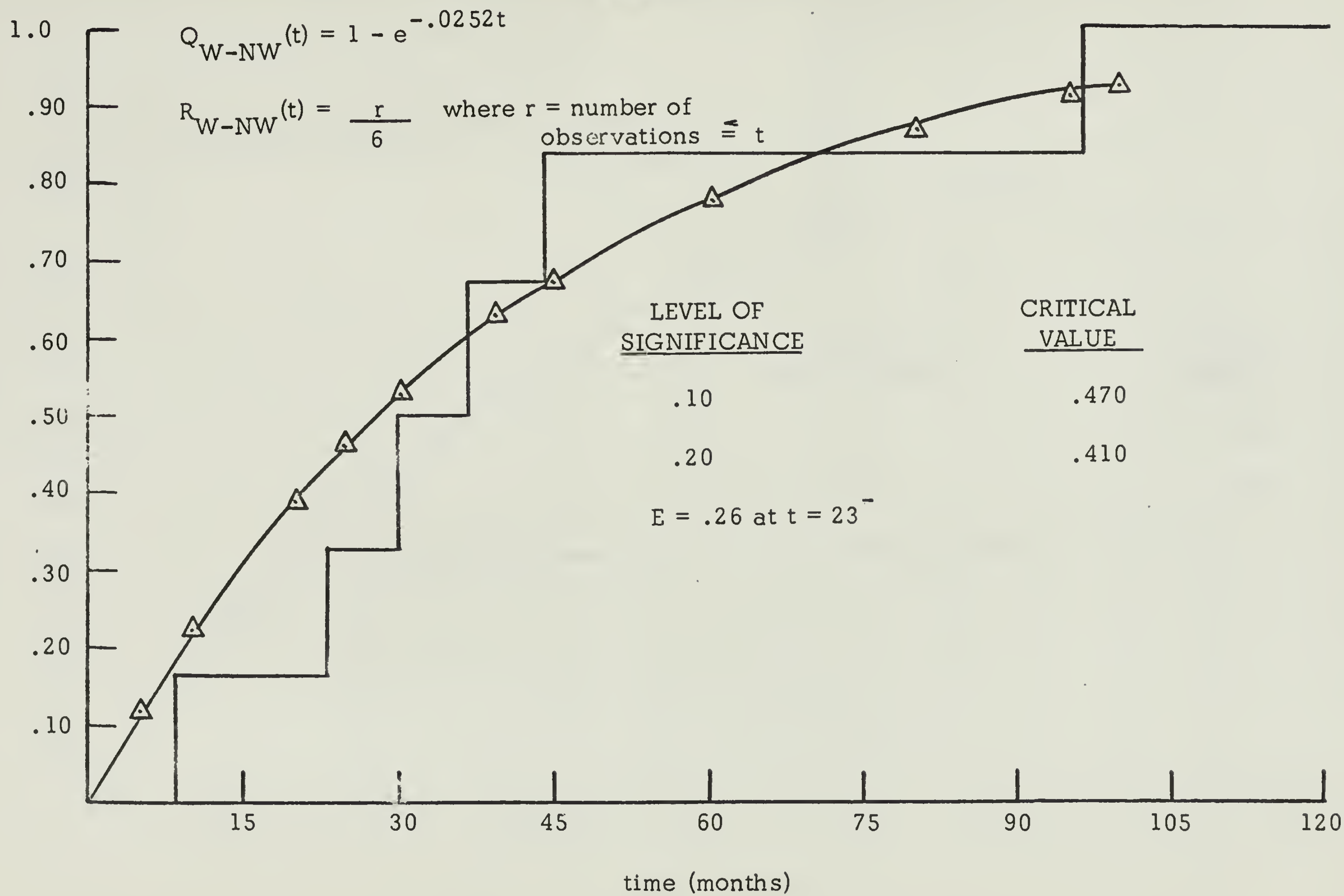


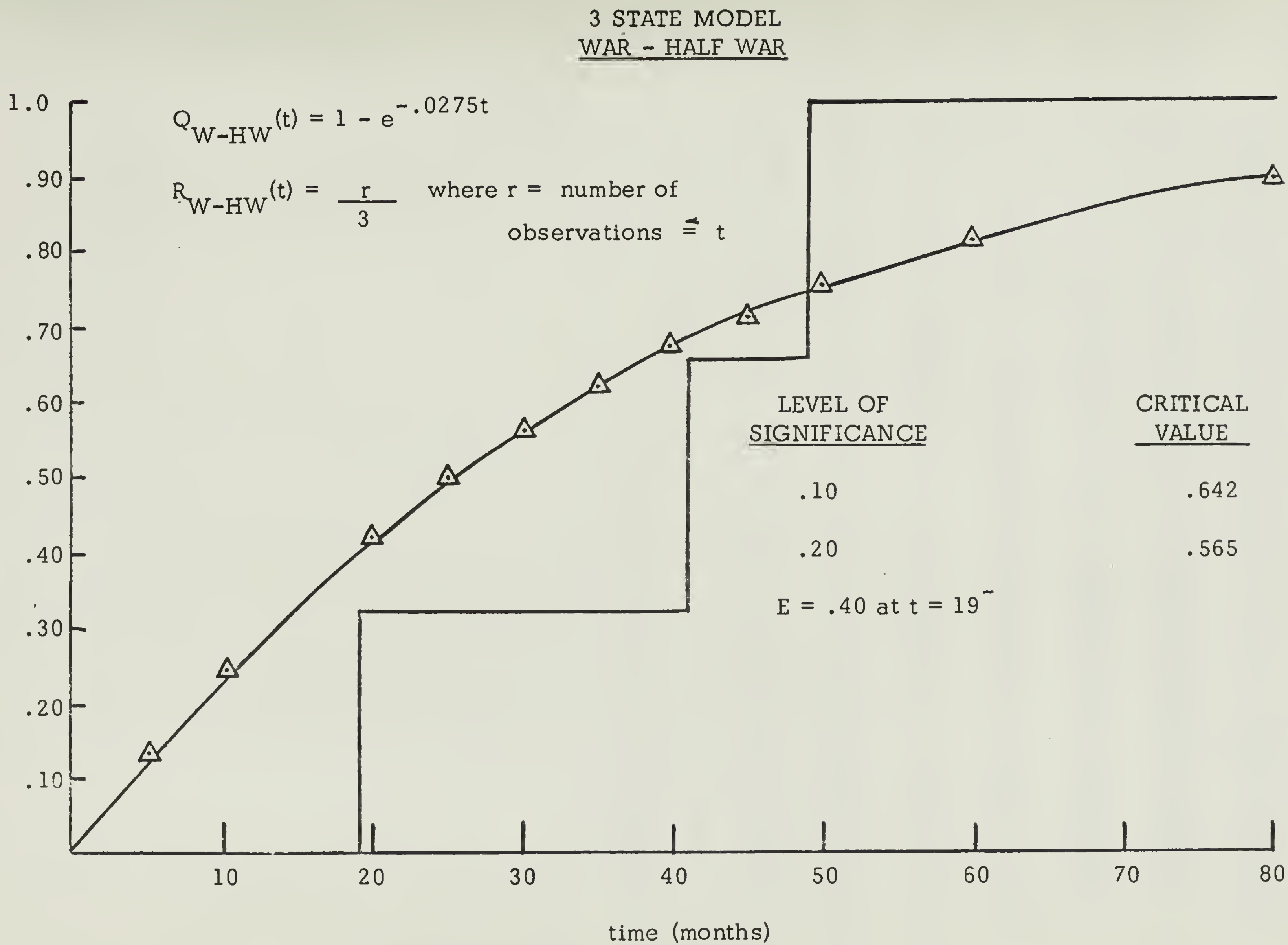
3 STATE MODEL
HALF WAR - WAR



3 STATE MODEL

WAR - NO WAR




3 STATE MODEL
WAR - HALF WAR
$Q_{W-HW}(t) = 1 - e^{-.0275t}$
$R_{W-HW}(t) = \frac{r}{3}$ where r = number of observations $\leq$ t
LEVEL OF SIGNIFICANCE
CRITICAL VALUE
.10
.642
.20
.565
E = .40 at t = 19^-
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
10
20
30
40
50
60
70
80
time (months)

LIST OF REFERENCES

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Security Classification

DOCUMENT CONTROL DATA - R & D

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1. ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

United States Military Involvements Since 1775 Modeled as a Markov Process

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Master's Thesis; September 1970

5. AUTHOR(S) *(First name, middle initial, last name)*

Richard Roland Pariseau

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1970	65	9

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Unclassified historical data concerning the United States involvement in military conflicts is used to model the behavioral patterns of the past as a Markov Process. The stochastic properties of the model are claculated and their usefulness as tools in the study of cost-effectiveness is discussed.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Markov Processes						
Cost Effectiveness						